Abstracts

Martin HAIRER

Title: *Solution Theory for Quasilinear Singular Stochastic PDEs*

Abstract. We give a construction allowing to obtain local renormalized solutions to general quasilinear stochastic PDEs within the theory of regularity structures, thus greatly generalizing the recent results of Gubinelli, Bailleul, Otto, Weber, etc. Loosely speaking, our construction covers quasilinear variants of all classes of equations to which the general semilinear solution theory recently developed in a series of works applies. This includes in particular one-dimensional systems with KPZ-type nonlinearities driven by space-time white noise. The main feature of our construction is that it allows to exploit a number of existing results developed for the semilinear case, so that the argument is relatively compact. This is joint work with M. Gerencsér.

Laure COUTIN

Title: *Some Results on Uniqueness for RDE Driven by Fractional Brownian Motion*

Abstract. First, we obtain a representation formula for some functionals of fractional Brownian motion. Then, using a previous work of R. Catellier and M. Gubinelli, we obtain some uniqueness results for additive RDE with respect to fractional Brownian motion.

Elisa ALOS

Title: *On Fractional Volatility Models*

Abstract. In this work we use Malliavin calculus techniques to study the short-time behavior of the at-the-money implied volatility, for correlated stochastic volatility models. Our analysis does not need the volatility to be a Markovian process. This approach gives us an interesting tool in modeling. In particular, we will see how the obtained results suggest, in order to reproduce short-time empirical data, considering fractional volatilities with Hurst parameter $H < 1/2$.

Paul GASSIAT

Title: *Existence of Densities for the Dynamic Φ^3_4 Model*

Abstract. Many nonlinear stochastic PDEs arising in statistical mechanics are ill-posed in the sense that one cannot give a canonical meaning to the nonlinearity. Nevertheless, Martin Hairer's theory of regularity structures provides us with a good notion of solution for a large class of such equations. An important example is given by the 3D stochastic quantization equation

$$(\partial_t - \Delta)u = -u^3 + Cu + \xi,$$

where ξ is space-time white noise. In this talk, we show that finite-dimensional (space-time) projections of the solution u have laws which are absolutely continuous with respect to Lebesgue's measure. Our proof is based on combining Malliavin calculus and tools from regularity structures theory. Interestingly, the result still holds if the noise is degenerate in some directions, as long as it is sufficiently rough on small scales. Based on a joint work with Cyril Labbé (Paris-Dauphine).

Alexandre RICHARD

Title: *Convergence to Equilibrium for Gaussian Driven SDEs*

Abstract. In this talk, I will present results on the convergence to the stationary regime for Stochastic Differential Equations driven by an additive Gaussian noise and evolving in a semi-contractive environment, i.e. when the drift is only contractive out of a compact set but does not have repulsive regions. In this setting, a sub-exponential bound is obtained on the rate of convergence to equilibrium in Wasserstein distance, and a similar bound in total variation distance is then derived. This is based on a joint work with F. Panloup.

Hélène HALCONRUY

Title: *Malliavin Calculus for Independent Random Variables*

Abstract. After years of development Malliavin calculus has reached a certain maturity with, in particular, resulted theories for Gaussian processes and Poisson point processes. In a work led in collaboration with Laurent Decreusefond, we construct a Malliavin framework for independent random variables not necessarily of the same distribution. This construction is a generalization of the known structure for Bernoulli independent random variables. A second motivation of this work has been to establish, within this formalism, convergence results using the Stein's method, and more specifically the approach of Malliavin integration by parts worked out by Nourdin and Peccati. In this talk, after having described the tools (definitions of the gradient, divergence, operator number) developed in this framework, will be presented, one of our results: a Stein criterion for Gaussian and Gamma approximations.

Antoine BRAULT

Title: *Differential Inclusions Perturbed by Rough Paths*

Abstract. A differential inclusion is an ordinary differential equation where the vector field is a multivalued function. The existence of solution depends on the regularity of the vector field and the convexity of its values. In this talk, we study a differential inclusion perturbed by a rough path. We prove the existence of solution without a convexity hypothesis by a fixed-point approach.

Laurent DECREUSEFOND

Title: *Mini Course on Malliavin Calculus*

Abstract. We present an introduction to the main concepts and proofs of Malliavin calculus. We define the notions of gradient and divergence at the same time for the Brownian motion and for fractional Brownian motions as they both come naturally from the Wiener space structure which is similar for these processes. We then show the basic similarities and difference with the stochastic calculus of variations for Poisson process. A new framework is then introduced for family of independent random variables. The Dirichlet form for Wiener and Poisson spaces can be recovered

by limiting procedures starting from this newly defined structure. We then show that how these elements interplay in the Stein-Malliavin-Dirichlet method.

Antoine LEJAY

Title: *A Short Introduction to Rough Paths*

Abstract. The theory of rough paths allows one to define integrals and differential equations driven by irregular signals at the price of extending the latter to enhanced paths called rough paths. Developed first by T. Lyons twenty years ago, this theory works well for defining pathwise integrals against Brownian motions, fractional Brownian motions and many other stochastic processes. In this short course, we present the main concepts behind the theory of rough paths, mainly the various forms of the so-called sewing lemma.

Ivan NOURDIN

Title: *Asymptotic Behavior of Large Gaussian Correlated Wishart Matrices*

Abstract. We consider high-dimensional Wishart matrices, associated with a rectangular random matrix X of size $n \times d$, whose entries are jointly Gaussian and correlated. Our main focus is on the case where the rows of X are independent copies of a n-dimensional stationary centered Gaussian vector of correlation function s. When s is 4/3 integrable, we show that a proper normalization of $d^{-1}XX^T$ is close in Wasserstein distance to the corresponding Gaussian ensemble as long as d is much larger than n^3. We also investigate the case where s is the correlation function associated with the fractional Brownian noise of parameter H. This example is very rich, as it gives rise to a great variety of phenomena with very different natures, depending on how H is located with respect to 1/2, 5/8 and 3/4. Notably, when $H > 3/4$, our study highlights a new probabilistic object, which we have decided to call the Rosenblatt-Wishart matrix. Our approach crucially rely on the fact that the entries of the Wishart matrices we are dealing with are double Wiener-Itô integrals, allowing us to make use of multivariate bounds arising from the Malliavin-Stein method and related ideas. This is joint work with Guangqu Zheng (Melbourne).

Marie KRATZ

Title: *On the Regularity of Time Occupation Functional for Gaussian Processes*

Abstract. We are interested in the order of Sobolev space $\mathbb{D}_p^\alpha$ of which the time occupational functional

$$\int_0^t \delta_x(X_s) f(\dot{X}_s) ds$$

belongs to, where $f : \mathbb{R} \to \mathbb{R}$ is a Borel measurable function and $X = \{X_t\}_{t \geqslant 0}$ is a centered stationary Gaussian process satisfying some conditions, to be discussed. This presentation is based on a study by M. Kratz (2000) and an ongoing joint work with Takafumi Amaba.

Aurélien DEYA

Title: *Rough Integration with Respect to Non-Commutative Processes*

Abstract. The non-commutative probability theory offers a privileged framework to study the asymptotic spectral behavior of random matrices whose size tends to infinity. Any (standard) Gaussian process admits a direct analog in the non-commutative setting, which in particular gives rise to the definition of the non-commutative Brownian motion, or more generally the non-commutative fractional Brownian motion of Hurst index H in (0,1).
In order to consolidate the correspondence between the standard and non-commutative worlds, it is then natural to wonder whether stochastic (or rough-path) integration methods can be extended to the non-commutative framework. This gives us another opportunity to test the flexibility of the rough-path principles and study (for instance) how the standard notion of Lévy area can be efficiently adapted in a non-commutative algebra setting.

Rémi CATELLIER

Title: *Mean-Field Rough Differential Equations*

Abstract. We provide in this work a robust solution theory for random rough differential equations of mean field type, with mean field interaction in both the drift and diffusivity. Propagation of chaos results for large systems of interacting rough differential equations are obtained as a consequence, with explicit optimal convergence rate. The development of these results requires the introduction of a new rough path-like setting and an associated notion of controlled path. We use crucially Lions' approach to differential calculus on Wasserstein space along the way.

Henri ELAD ALTMAN

Title: *Renormalization Phenomena in Stochastic PDEs with Reflection*

Abstract. In the early 2000s, Zambotti introduced a family of SPDEs parametrized by a real number d larger or equal to 3, having as invariant measure the law of the d-dimensional Bessel bridge. A long-standing open problem is the construction of such processes when d is smaller than 3. A partial answer is provided by the study of integration by parts formulae (IbPF) satisfied by the laws of Bessel bridges. Such formulae were recently obtained also for bridges of dimension less than 3, thus allowing to conjecture the structure of corresponding SPDEs. In my talk I will discuss these formulae, and explain the renormalization procedure they involve. Of particular interest are the formulae for integer values of d, which may be written using a KPZ type renormalization, and are related to infinite-dimensional Tanaka formulas.

Carlo BELLINGERI

Title: *Stochastic Heat Equation and Rough Paths*

Abstract. In this talk we will present how the rough path formalism can be applied to deduce new change of variable formulae for the time evolution of the additive stochastic heat equation. This is a Gaussian process with a.s. continuous path which is not a semimartingale. Compared to the previous literature, the resulting formulae will hold always a.s. and not only in law.

Blanka HORVATH

Title: *Functional Central Limit Theorems for Rough Volatility*

Abstract. We extend Donsker's approximation of Brownian motion to fractional Brownian motion with Hurst exponent $H \in (0,1)$ and to Volterra-like processes. Some of the most relevant consequences of our *rough Donsker (rDonsker) Theorem* are convergence results for discrete approximations of a large class of rough models. This justifies the validity of simple and easy-to-implement Monte-Carlo methods, for which we provide detailed numerical recipes. We test these against the current benchmark Hybrid scheme and find remarkable agreement (for a large range of values of *H*). This rDonsker Theorem further provides a weak convergence proof for the Hybrid scheme itself, and allows to construct binomial trees for rough volatility models, the first available scheme (in the rough volatility context) for early exercise options such as American or Bermudan.

Alexandre BROUSTE

Title: *Parametric Estimation at High-Frequency*

Abstract. Asymptotic efficiency of the sequence of maximum likelihood estimators is considered in statistical experiments implying the fractional Gaussian noise or symmetric stable random variables observed at high-frequency. Likelihood ratio hypothesis tests are also studied with an application to oil price modeling.

Francesco RUSSO

Title: *Recent Developments in Stochastic Calculus via Regularizations with Jumps and Applications to BSDEs*

Abstract. The aim of this talk consists in mentioning recent developments about stochastic calculus via regularizations for jump processes. We recall that a *weak Dirichlet process* X with respect to a given underlying filtration is the sum of a local martingale and a process A such that $[A, N] = 0$ for every continuous local martingale. We introduce the notion of special weak Dirichlet process; whenever such a process is a semimartingale, then it is a special semimartingale. We will provide conditions on a function $u : [0,T] \times \mathbb{R}^d \to \mathbb{R}$ and on an adapted cadlag process S such that $u(t, S_t)$ is a special weak Dirichlet process. Two applications will be discussed.

1. The existence of a solution to a (strong) solution of a BSDEs with distributional driver, with underlying Brownian filtration (with Elena Issoglio, Leeds).

2. Consider the case a BSDE driven by a random measure: a solution is a triplet (Y, Z, K) where K is a random field. The function $u(s, x) := Y_s^{s,x}$ is deterministic. If u has some minimal regularity, the calculus will allow to link Z, K to u (with Elena Bandini, Milano Bicocca).